CIRCLE THEOREMS

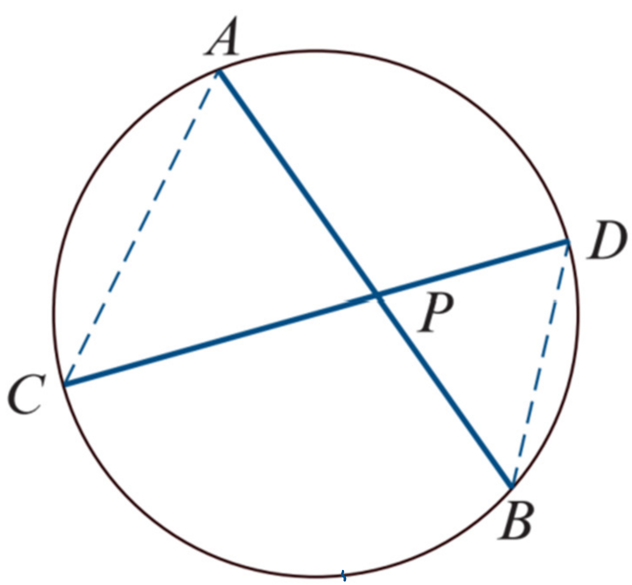
Year 11 Mathematics Specialist

Tangents and Secants

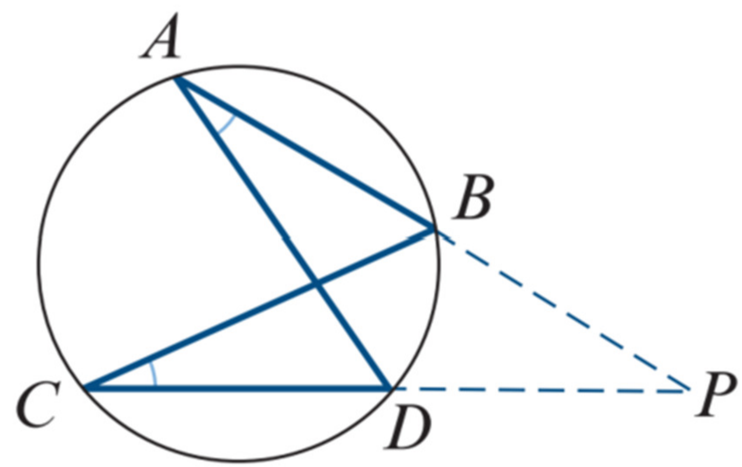
8C Chords in circles

Theorem 8

If AB and CD are two chords of a circle that cut at a point P (which may be inside or outside the circle), then $PA \cdot PB = PC \cdot PD$.



case 1

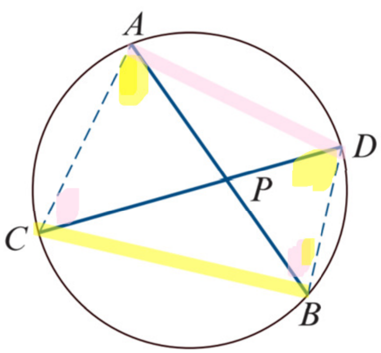


case 2

Theorem 8

If AB and CD are two chords of a circle that cut at a point P (which may be inside or outside the circle), then $PA \cdot PB = PC \cdot PD$.

Case 1: P is inside the circle



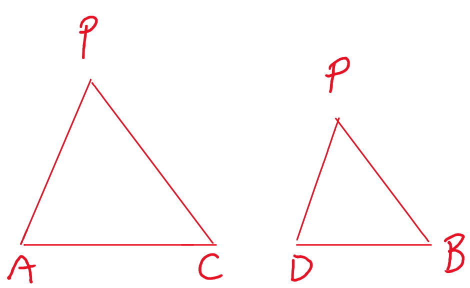
consider $\triangle APC$ and $\triangle DPB$

$\angle APC = \angle DPB$ (vertically opposite)

$\angle CAB = \angle BDC$ (angles in the same segment)

$\angle ACD = \angle ABD$ →

$\therefore \triangle APC \sim \triangle DPB$ (AAA)



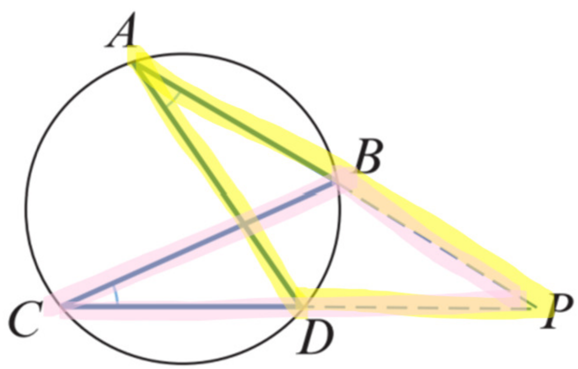
$$\frac{PA}{PD} = \frac{PC}{PB}$$

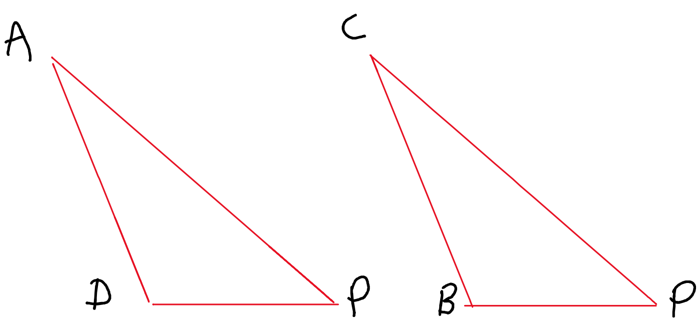
$\therefore PA \times PB = PC \times PD$

Theorem 8

If AB and CD are two chords of a circle that cut at a point P (which may be inside or outside the circle), then $PA \cdot PB = PC \cdot PD$.

Case 2: P is outside the circle





consider $\triangle APD$ and $\triangle CPB$

$\angle APD = \angle CPB$ (common angle)

$\angle BAD = \angle BCD$ (angles in same segment)

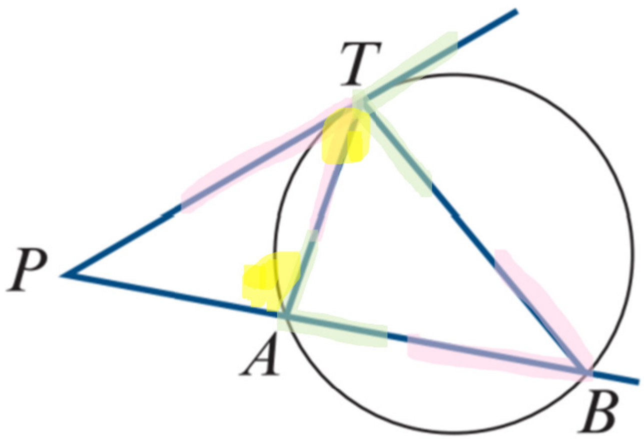
$\therefore \triangle APD \sim \triangle CPB$

$$\frac{PA}{PC} = \frac{PD}{PB}$$

$\therefore PA \times PB = PC \times PD$

Theorem 9

If P is a point outside a circle and T, A, B are points on the circle such that PT is a tangent and PAB is a secant, then $PT^2 = PA \cdot PB$.



consider $\triangle PAT$ and $\triangle PTB$

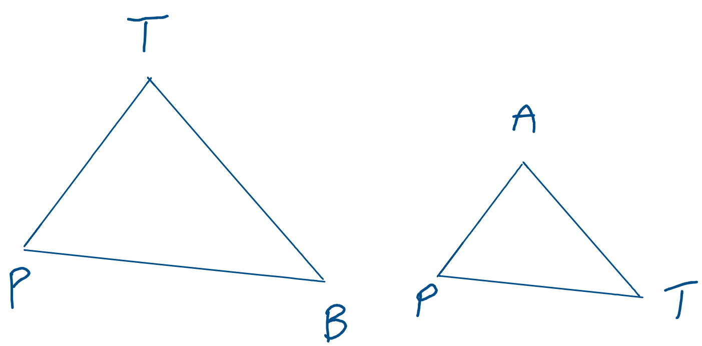
$\angle PTA = \angle ABT$ (alternate segment theorem)

$\angle PAT = \angle PTB$

$\therefore \triangle PAT \sim \triangle PTB$ (AAA)

$$\frac{PA}{PT} = \frac{PT}{PB}$$

$\therefore PT^2 = PA \times PB$



Example 7

The arch of a bridge is to be in the form of an arc of a circle. The span of the bridge is to be 25 m and the height in the middle 2 m. Find the radius of the circle.

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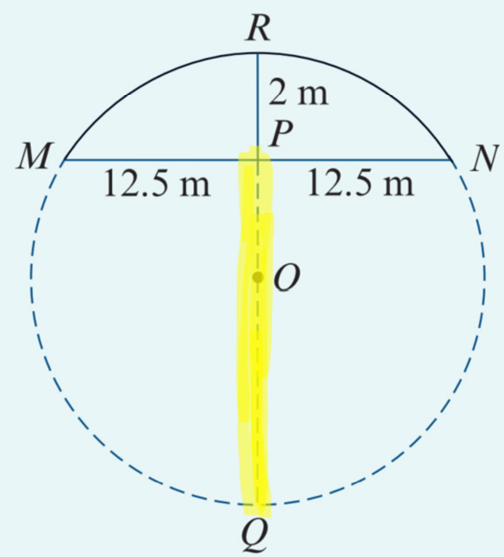
Solution

Let r be the radius of the circle. Then $PQ = 2r - 2$.



Use Theorem 8 with the chords RQ and MN:

$$RP \cdot PQ = MP \cdot PN$$

Therefore

$$2PQ = 12.5^2$$

$$PQ = \frac{12.5^2}{2}$$

$$2r - 2 = \frac{12.5^2}{2} \qquad \text{as } PQ = 2r - 2$$

$$\therefore \quad r = \frac{1}{2}\left(\frac{12.5^2}{2} + 2\right)$$

$$= \frac{641}{16} \text{ m}$$

Example 8

Let A be any point inside a circle with radius r and centre O. Show that, if CD is a chord through A, then $CA \cdot AD = r^2 - OA^2$.

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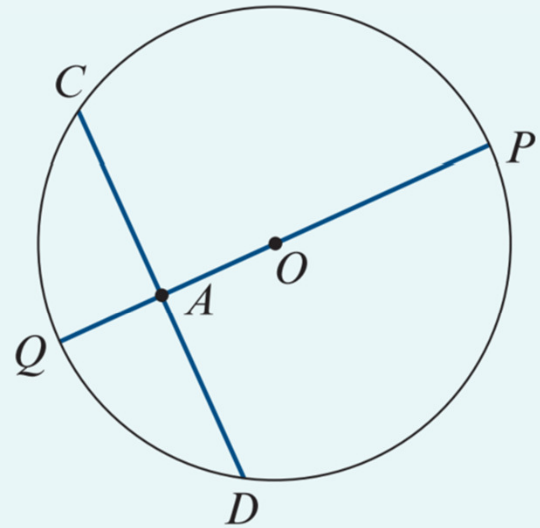
Solution

Let PQ be a diameter through A as shown.



By Theorem 8:

$$CA \cdot AD = QA \cdot AP$$

Since $QA = r - OA$ and $AP = r + OA$, this gives

$$\begin{aligned} CA \cdot AD &= (r - OA)(r + OA) \\ &= r^2 - OA^2 \end{aligned}$$